

Hong Kong Mathematics Olympiad (1992 – 93)

Heat Event (Group)

香港数学竞赛(1992 – 93)

初赛项目 (团体)

1. A car  $P$  is  $10\sqrt{2}$  km north of another car  $Q$ . The two cars start to move at the same time with  $P$  moving south-east at 4 km/h and  $Q$  moving north-east at 3 km/h. Find their smallest distance of separation in km.

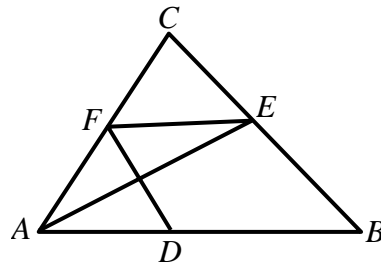
一汽车  $P$  位于另一汽车  $Q$  以北  $10\sqrt{2}$  km。两车同时起步，其中  $P$  以 4 km/h 速度向东南方走， $Q$  则以 3 km/h 速度向东北方走。求两车最接近时的距离并以 km 表之。

2. If  $\alpha, \beta$  are the roots of the equation  $x^2 - 3x - 3 = 0$ , find  $\alpha^3 + 12\beta$ .

若  $\alpha, \beta$  为方程  $x^2 - 3x - 3 = 0$  的两根，求  $\alpha^3 + 12\beta$ 。

3. As shown in figure 1, the area of  $\triangle ABC$  is 10.  $D, E, F$  are points on  $AB, BC$  and  $CA$  respectively such that  $AD : DB = 2 : 3$ , and area of  $\triangle ABE =$  area of quadrilateral  $BEFD$ . Find the area of  $\triangle ABE$ .

在图 1 中，三角形  $ABC$  的面积为 10。 $D, E$  及  $F$  分别为  $AB, BC$  及  $CA$  上的点且满足  $AD : DB = 2 : 3$ ，且  $\triangle ABE$  的面积 = 四边形  $BEFD$  的面积。求  $\triangle ABE$  的面积。



(Figure 1) (图 1)

4. What is the maximum number of regions produced by drawing 20 straight lines on a plane?

在一平面上画 20 条直线，最多可将平面分成几个区域？

5. The product of 4 consecutive positive integers is 3024. Find the largest integer among the four.

若四个连续正整数的乘积为 3024，求其中最大的一个。

6. Find the sum of all real roots of the equation  $(x+2)(x+3)(x+4)(x+5) = 3$ .

求方程  $(x+2)(x+3)(x+4)(x+5) = 3$  的实根的总和。

7. If  $a$  is an integer and  $a^7 = 8031810176$ , find the value of  $a$ .

若  $a$  为一整数，且  $a^7 = 8031810176$ ，求  $a$  的值。

8. If  $x$  and  $y$  are real numbers satisfying

$$\begin{cases} x^2 - xy + y^2 - 3x - 3y = 1 \\ xy = 1 \end{cases}$$

and  $x > y > 0$ , find the value of  $x$ .

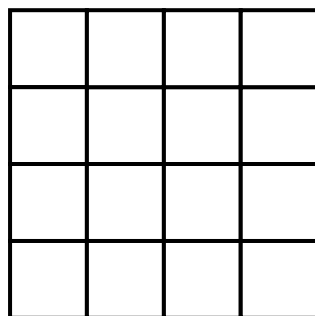
若  $x$  及  $y$  为实数，且

$$\begin{cases} x^2 - xy + y^2 - 3x - 3y = 1 \\ xy = 1 \end{cases}$$

及  $x > y > 0$ ，求  $x$  的值。

9. Each side of a square is divided into four equal parts and straight lines are joined as shown in figure 2. Find the number of rectangles which are not squares.

一正方形的每边被直线均分为四份，如图 2。求非正方形的长方形数目。



(Figure 2) (图 2)

10. If  $0^\circ \leq \theta \leq 90^\circ$  and  $\cos \theta - \sin \theta = \frac{\sqrt{5}}{3}$ , find the value of  $\cos \theta + \sin \theta$ .

若  $0^\circ \leq \theta \leq 90^\circ$ ，且  $\cos \theta - \sin \theta = \frac{\sqrt{5}}{3}$ ，求  $\cos \theta + \sin \theta$  的值。